

Gluonic plasma dominated early universe within fluid QCD

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Abstract

An early universe dominated by gluonic plasma just before the hadronization is investigated. Using the fluid QCD framework, it is shown that during the QGP–hadron transition era the universe tends expanding exponentially with 40 e-fold scale factor, while the equation of state numerically yields $P \sim \frac{1}{5}\rho$. It is argued that the scenario supports the rather matter dominated hadronization phase, and agrees with the current flat and homogeneous universe.

Keywords : quark-gluon-plasma, fluid QCD, early universe

1 Introduction

Recent experiments on heavy-ion collisions show a strong indication that hot dense deconfined phase of free quark and gluon, the so called quark-gluon plasma (QGP), is conjectured to be formed. The QGP state is also believed to exist at early universe before hadronization, which was very short after the inflation era. This scenario has been concluded through different approaches in many previous works. Some of them have been obtained within the quantum chromodynamics (QCD) theory using the lattice gauge calculation [1, 2]. Another calculations on QGP have been conducted based on the relativistic hydrodynamics approach [3, 4]. Within this framework, QGP could either be quark [3] or gluon [4] dominated matter. For quark dominated approach, a very small ratio of shear

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viscosity over entropy is required to get a good fit of the spectra of transverse momentum, energy density distribution and other physical observables obtained from experiments [5, 6, 7, 8, 9, 10]. On the other hand, the gluon dominated plasma is motivated by the discoveries of jet quenching in heavy-ion collision at RHIC indicating the shock waves in the form of March cone [11, 12]. These new understandings on QGP lead to many works exploring the QGP dominated early universe [13, 14, 15, 16, 17].

There are a series of phase or multi-phases transitions that came after the universe inflation. The present paper is focused on the QGP epoch before the hadronic phase. This could happen at the energy of 1 GeV ($\sim 10^{-6}$ s) after inflation. During this period, another matters like leptons are assumed to be much less dominated. The paper adopts the so-called fluid QCD model [18, 19] to deal with the equation of state, and subsequently to investigate the cosmological scale factor and Hubble parameter at the QGP-hadron transition time during the early universe.

The paper is organized as follows. First a brief introduction to the fluid QCD model is presented. Using the pressure and density yielded in the model, the cosmological field equation is derived in the subsequent section. Finally, the numerical analysis for Hubble parameter H and cosmological scale factor R are performed. Finally, the paper is concluded with summary and discussion.

2 Model

The model under consideration is briefly reviewed in this section. The model has been proposed to describe the gluonic plasma [18]. Considering the gluon dominated plasma, the model is governed by the standard non-Abelian lagrangian,

$$\mathcal{L} = -\frac{1}{4}S_{\mu\rho}^a S^{a\mu\rho} + g_s J_\mu^a U^{a\mu}, \quad (1)$$

where U_μ is the gauge vector field and g_s is the strong coupling constant. $J_\mu^a = \bar{Q}T^a\gamma^\mu Q$ and $S_{\mu\rho}^a = \partial_\mu U_\rho^a - \partial_\rho U_\mu^a + g_F f^{abc}U_\mu^b U_\rho^c$, where T^a is the SU(3) Gell-Mann matrices, and f^{abc} is the structure constant of SU(3) group and Q represents the quark (color) triplet.

In order to raise its fluidic behavior, it has been argued that U_μ^a should be rewritten in a particular form of relativistic velocity as $U_\mu^a = (U_0^a, \mathbf{U}^a) \equiv u_\mu^a \phi$ [18, 19]. Here, $u_\mu^a \equiv \gamma \mathbf{v}^a (1, \mathbf{v}^a)$ and $\gamma \mathbf{v}^a = (1 - |\mathbf{v}^a|^2)^{-1/2}$ with $\phi = \phi(x)$ is a dimension one scalar field to keep a correct dimension and should represent the field distribution. Equation of motion for a single gluon field from the above lagrangian has produced a general relativistic fluid equation [18]. This fact explains that a single gluonic field U_μ^a may behaves as a fluid at certain phase, and also as point particle at hadronic state with a polarization vector ϵ_μ in the conventional form of $U_\mu^a = \epsilon_\mu^a \phi$. This is considered as a kind of “phase transition” between fluidic and hadronic states. More recent details can be found in [20].

Using the lagrangian in Eq. (1), one can derive the energy momentum tensor for QGP through the energy momentum tensor density,

$$\mathcal{T} = \frac{2}{\sqrt{-g}} \frac{\delta \mathcal{L}_g}{\delta g^{\mu\nu}}, \quad (2)$$

which leads to [20],

$$\mathcal{T}_{\mu\nu} = [8g_s f_Q m_Q \phi + g_s^2 f_g^2 \phi^4] u_\mu u_\nu - [4g_s f_Q m_Q \phi - \frac{1}{4} g_s^2 f_g^2 \phi^4] g_{\mu\nu}, \quad (3)$$

for homogeneous color states, *i.e.* $U_\mu^a = U_\mu$ for all $a = 1, \dots, 8$. f_g is the factor of summed colored gluon states from the structure constant f^{abc} , while f_Q is the factor of summed colored quark states from $J_\mu^a U^{a\mu}$. The total energy momentum tensor $T_{\mu\nu}$ is obtained by integrating out the equation in term of total volume in the space-time under consideration, *i.e.* $T_{\mu\nu}$ is a result of bulk of gluons flow in the system. The quark current J_μ^a can be simply calculated by considering Dirac equation (EOM) of single colored quark (Q) or anti-quark (\bar{Q}) with 4-momentum p_μ [20]. Throughout the paper let us use the FRW metric. This means the model falls back to the perfect fluid without viscosity and heat conduction, and realizes isotropic and homogeneous universe. Therefore it plausible to take the gluon distribution which depends only on the time scale, $\phi = \phi(t)$.

Therefore, one can obtain the density and pressure in the model as follows [20],

$$P = \int \mathcal{P} d^4x = 4g_s f_Q m_Q \int \int \left(1 - \frac{g_s f_g^2}{16 f_Q m_Q} \phi^3 \right) \phi dt dV, \quad (4)$$

$$\rho = \int \mathcal{E} d^4x = 4g_s f_Q m_Q \int \int \left(1 + \frac{5g_s f_g^2}{16 f_Q m_Q} \phi^3 \right) \phi dt dV. \quad (5)$$

Here \mathcal{P} and \mathcal{E} denote the isotropic pressure and density for single fluid field.

Now we are ready to proceed further to perform numerical analysis based on these results.

3 Cosmological field equation

Integrating out the spatial dimension in FRW geometry, Eqs. (4) and (5) read,

$$P = \lambda \int R^3 \phi (1 - \delta \phi^3) dt \int \frac{r^2 dr}{\sqrt{1 - kr^2}}, \quad (6)$$

$$\rho = \lambda \int R^3 \phi (1 + \kappa \phi^3) dt \int \frac{r^2 dr}{\sqrt{1 - kr^2}}, \quad (7)$$

using $dV = (R^3 r^2 \sin\theta) / \sqrt{1 - kr^2} dr d\theta d\vartheta$. Further, taking the time derivative,

$$\dot{\rho} = \lambda R^3 \phi [1 + \kappa \phi^3] \int \frac{r^2 dr}{\sqrt{1 - kr^2}}, \quad (8)$$

where, $\kappa = (5g_s f_g^2) / (16 f_Q m_Q)$ and $\lambda = 16\pi g_s f_Q m_Q$.

Concerning that the FRW space-time is occupied by the plasma, from the cosmological field equations,

$$\ddot{R} = -\frac{4\pi G}{3} \left(\rho + \frac{3p}{c^2} \right) R + \frac{1}{3} \Lambda c^2 R, \quad (9)$$

$$\dot{R}^2 = \frac{8\pi G}{3} \rho R^2 + \frac{1}{3} \Lambda c^2 R^2, \quad (10)$$

and assuming the curvature $k \sim 0$ due to flat universe and tiny cosmological constant $\Lambda \sim 0$, one immediately obtains,

$$\dot{\rho} + \left(\rho + \frac{p}{c^2} \right) \frac{3\dot{R}}{R} = 0. \quad (11)$$

Substituting Eqs. (6),(7) and (8) into Eq. (11), and taking again its time derivative, one gets,

$$-\frac{\ddot{R}R}{\dot{R}^2}\phi(1+\kappa\phi^3) + \frac{R}{\dot{R}}\dot{\phi}(1+4\kappa\phi^3) + \phi[10 + (7\kappa - 3\delta)\phi^3] = 0, \quad (12)$$

where $\delta = \kappa/5$.

Furthermore the Hubble parameter, $H(t) = \dot{R}(t)/R(t)$, becomes,

$$-\frac{\dot{H}}{H^2}\phi(1+\kappa\phi^3) + \frac{1}{H}\dot{\phi}(1+4\kappa\phi^3) + \phi[9 + (6\kappa - 3\delta)\phi^3] = 0. \quad (13)$$

Putting $\nu = 1/H$, Eq. (13) is rewritten to be,

$$\dot{\nu} + \frac{\dot{\phi}(1+4\kappa\phi^3)}{\phi(1+\kappa\phi^3)}\nu = -\frac{9 + (6\kappa - 3\delta)\phi^3}{1 + \kappa\phi^3}. \quad (14)$$

This is nothing else than the Bernoulli form of differential equation.

Now, taking the gluon field as a plane wave representing a free particle, it should have a form $\phi(t) = e^{iEt}$. Substituting it into Eq. (13), through a standard mathematical procedure the solution for ν is,

$$\nu(t) = \frac{36i + 4Ee^{-iE\Delta t} + i\mu e^{3iE\Delta t}}{4E + 4\kappa E e^{3iE\Delta t}}, \quad (15)$$

with $\mu = 6\kappa - 3\delta$. Expanding the imaginary terms using $e^{iE\Delta t} = \cos(E\Delta t) + i\sin(E\Delta t)$, one can separate the real and imaginary parts as,

$$\nu_r = \frac{\cos(E\Delta t) + \kappa \cos(4E\Delta t) + (36\kappa - \mu)/(4E) \sin(3E\Delta t)}{1 + \kappa^2 + 2\kappa \cos(3E\Delta t)}, \quad (16)$$

$$\nu_i = \frac{-\sin(E\Delta t) - \kappa \sin(4E\Delta t) + (36 + \mu)/(4E) \cos(3E\Delta t) + (36 + \mu\kappa)/(4E)}{1 + \kappa^2 + 2\kappa \cos(3E\Delta t)}. \quad (17)$$

These results lead to,

$$H_r = \frac{xz}{x^2 + y^2} \quad \text{and} \quad H_i = \frac{yz}{x^2 + y^2}, \quad (18)$$

which yields,

$$|H| = \frac{z}{\sqrt{x^2 + y^2}}. \quad (19)$$

Here,

$$x = \cos(E\Delta t) + \kappa \cos(4E\Delta t) + \frac{36\kappa - \mu}{4E} \sin(3E\Delta t), \quad (20)$$

$$y = -\sin(E\Delta t) - \kappa \sin(4E\Delta t) + \frac{36 + \mu}{4E} \cos(3E\Delta t) + \frac{36 + \mu\kappa}{4E}, \quad (21)$$

$$z = 1 + \kappa^2 + 2\kappa \cos(3E\Delta t). \quad (22)$$

The result is depicted in Fig. 1 for the QGP-hadron epoch using typical values : $f_Q = 6$, $f_g = 17.5$, $m_Q = 0.01$ GeV and $G_s = 1$. The duration of hadronization is put at the order of $\Delta t \sim 1$ eV to assure the radiated QGP can reach the whole hadronization radii. It should be remarked that the Hubble constant at QGP-hadron transition, namely $E \sim 100$ GeV, is found to be $H \sim 74.1 \pm 2.9 \text{ s}^{-1} \text{ km Mpc}^{-1}$.

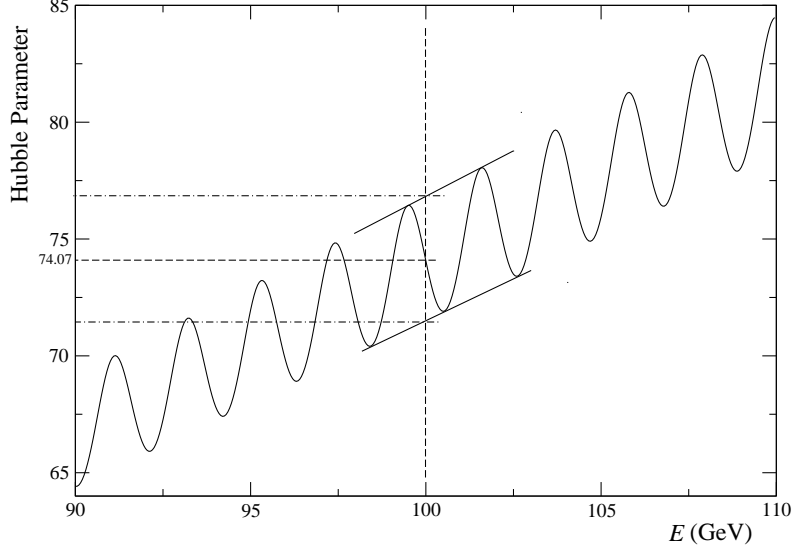


Figure 1: The Hubble parameter as a function of energy.

Finally, let us investigate the cosmological scale factor R which is another observable directly related to the Hubble parameter. Making use of the relation $H = \dot{R}/R = 1/R dR/dt$, one obtains

$$R = e^{\int H dt} . \quad (23)$$

Using the preceding values for all parameters, it yields $R \sim e^{42.8}$, *i.e.* around 40 e-folding.

After obtaining the Hubble constant and cosmological scale factor at hadronization epoch, one may continue performing the calculation for the QGP–hadron transition radii. From Eqs. (4), (5) and (11),

$$\rho = \frac{-i\lambda R^3 r^3}{3E} \left(e^{iE\Delta t} + \frac{\kappa}{4} e^{4iE\Delta t} \right) , \quad (24)$$

$$P = \frac{-i\lambda R^3 r^3}{3E} \left(e^{iE\Delta t} - \frac{\delta}{4} e^{4iE\Delta t} \right) . \quad (25)$$

After performing straightforward rearrangement, Eq. (10) reads,

$$H^2 = \frac{8\pi G}{3} \rho R^2 , \quad (26)$$

which leads to,

$$r^3 = \frac{-2i\pi G \lambda R^3 r^3}{9E} (4e^{iE\Delta t} + \kappa e^{4iE\Delta t}) . \quad (27)$$

For the above mentioned values for parameters, the hadronization radii is $|r| \sim 2.9 \times 10^{-6}$ eV, or ~ 2.5 fm, while the density–pressure ratio is $|\rho|/|P| \sim 5$ accordingly.

4 Summary and Discussion

The study of gluon dominated early universe during QGP–hadronization epoch has been elaborated. The cosmological field equation in the case of isotropic and homogeneous FRW space-time has been investigated. It has been found that the Hubble parameter, $H \sim 74.1 \pm 2.9 \text{ s}^{-1} \text{ km Mpc}^{-1}$ which should be valid at QGP–hadron transition era. It is remarkable that the value coincides to its current value. This means it justifies the consensus that the Hubble constant should be considered as a "constant" rather than a parameter. This also supports previous results from several cosmologist groups who had found that the correct Hubble constant stays at $100h$, with $h \sim 0.7 \text{ s}^{-1} \text{ km Mpc}^{-1}$ [21, 22].

Meanwhile, in the present paper the scale factor could reach to 40 e-fold. It indicates that during the radiation-matter transition era, the universe still expanded exponentially. Though the rate is much lower than the inflation prior to the QGP era. It agrees with some of the early universe conjectures which predicted the inflation should happen during the time. Referring to previous studies on inflationary cosmology, the current flatness and homogeneity of our universe could be realized only when there was an inflation of 17 to 68 e-folding, corresponding to $h = 0.7$ [23].

It should also be noted that the present model predicted that $|P| \sim 1/5|\rho|$. This suggests that at the QGP–hadron transition era the matter already started dominating the universe. Thereafter the ratio $|P|/|\rho|$ is getting smaller along the hadronization, and approaching zero when our universe is fully dominated by matter. The present work at least confirms that the equation of state is just suitable to describe the relation between pressure and density for the cosmic fluid at a time within hadronization epoch.

Finally, the study revealed observables which should be accessible at the RHIC and LHC experiments reproducing the QGP–hadron transition epoch at the early universe. Such experiments are in the future expected to clarify the time scale of hadronization, radius of interaction, pressure and energy density during the epoch.

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